Long String Scattering in $c = 1$ String Theory

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with B. Balthazar, X. Yin [1705.07151,1810.07233]

Strings 2019
July 9

$c = 1$ String Theory

Worldsheet theory

$$\underset{\text{(time)}}{X^0} \quad + \quad \underset{\text{(space)}}{c = 25 \text{ Liouville CFT}} \quad + \quad b, c \text{ ghosts}$$

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Liouville CFT

- spectrum: scalar Virasoro primaries

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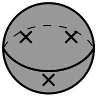
- OPE coefficients: given by the DOZZ formula



$$\mathcal{C}(P_1, P_2, P_3) = \text{Known}$$

This is all the data you need to compute any correlation function in Liouville theory.

$c = 1$ String Theory

BRST cohomology representatives

$$\mathcal{V}^{\pm}_{\omega} = g_s : e^{\pm i\omega X^0} : V_{P=\omega/2}$$

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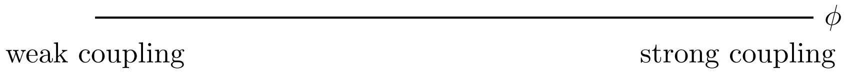
$$S_L[\phi] = \frac{1}{4\pi} \int_{\Sigma} d^2\sigma \sqrt{g} \left(g^{mn} \partial_m \phi \partial_n \phi + 2R\phi + 4\pi\mu e^{2\phi} \right)$$

Spacetime picture:



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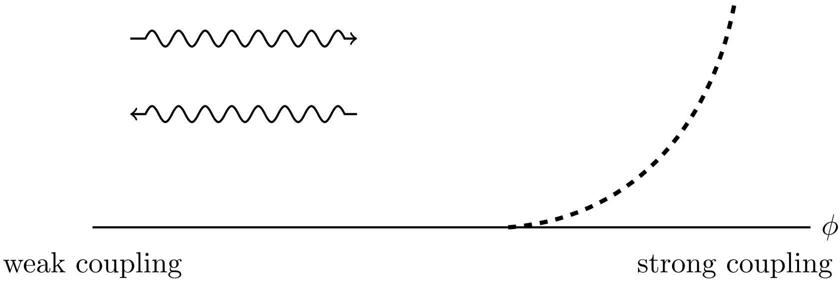

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Conformal boundary conditions in Liouville theory.

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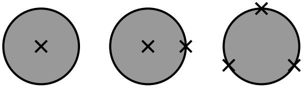

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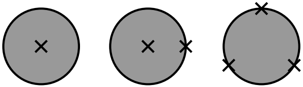

Known

Open strings on FZZT branes are represented by
(Neumann boundary conditions for X^0)

$$\Psi_\omega^{s_1,s_2\ \pm} = g_o \,{}^*_*\, e^{\pm i\omega X^0} \,{}^*_*\, \psi_{P=\omega}^{s_1,s_2}$$

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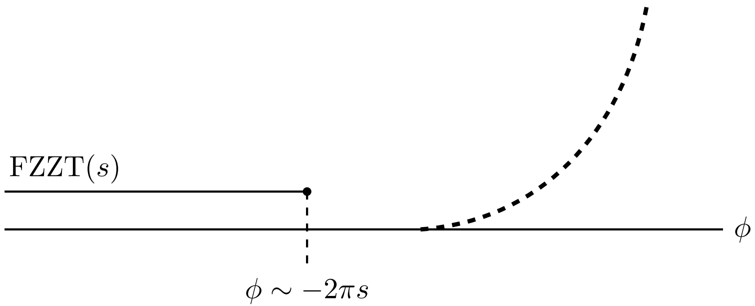
Spacetime interpretation:



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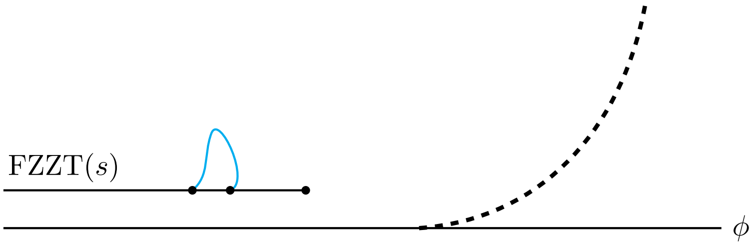
Today we will study the dual of the FZZT brane, indirectly.



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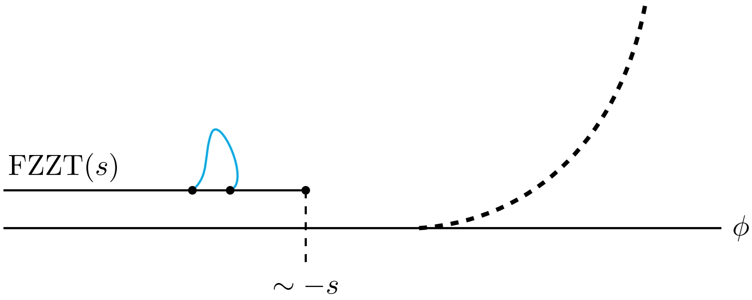

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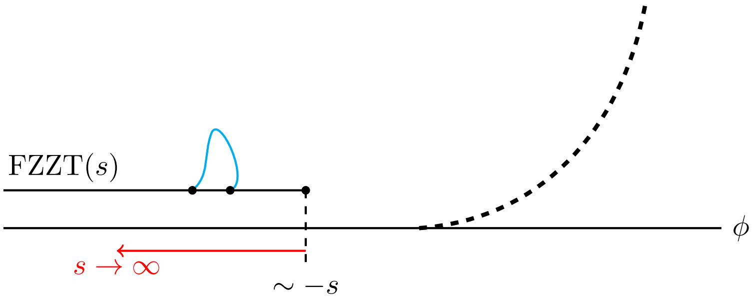

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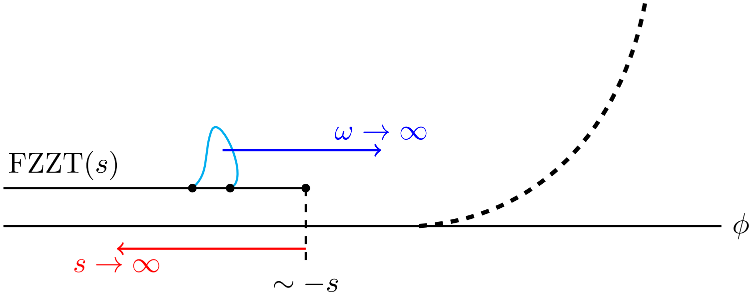

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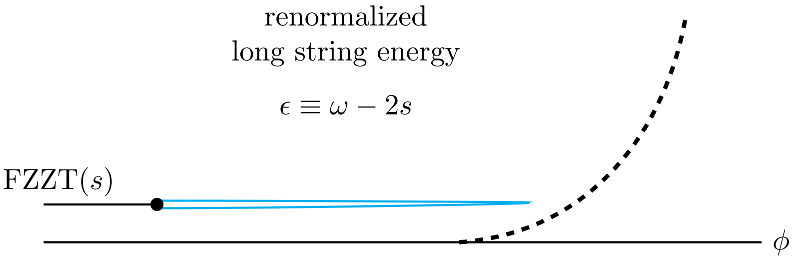

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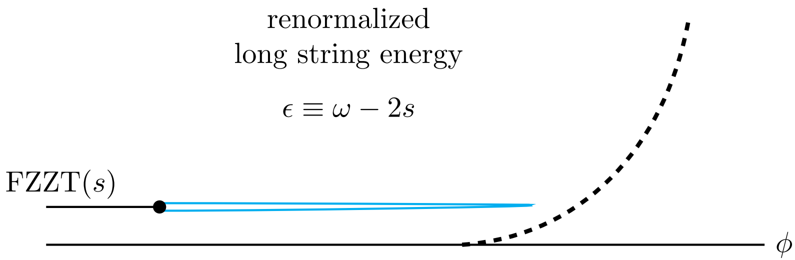

Proposal [Maldacena]

long strings	$\longleftrightarrow$	states in adjoint sector of MQM

$L \to L + C$ amplitude from the worldsheet

Amplitude describing the decay of a long string by emitting a closed string:

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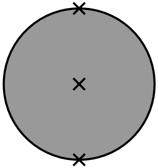

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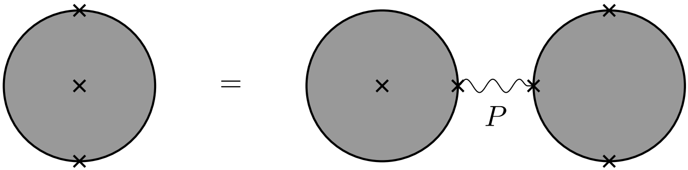

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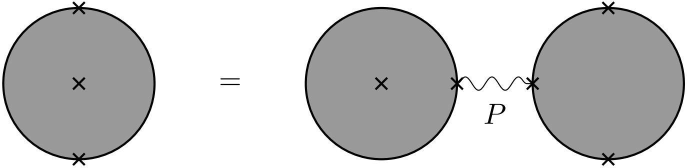

$$\lim_{\substack{s\to\infty \\ \epsilon_i \text{ fixed}}} \int (\text{moduli}) \quad \int_0^\infty dP \quad \mathcal{R}^s \quad \times \quad C^{s,s,s} \quad \times \quad \begin{matrix}\text{conformal} \\ \text{block}\end{matrix}$$

Matrix Model

Quantum mechanics of an $N \times N$ Hermitian matrix M, with Hamiltonian

$$H = \mathrm{Tr}\left[\frac{1}{2}P^2 - \frac{1}{2}M^2\right]$$

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Following standard procedure, going to polar coordinates $M = \Omega^\dagger \Lambda \Omega$, one obtains

$$H' = \underbrace{\frac{1}{2}\sum_{i=1}^{N}\left[-\frac{\partial^2}{\partial\lambda_i^2} - \lambda_i^2 + 2\mu\right]}_{\text{singlet}} + \underbrace{\frac{1}{2}\sum_{i\neq j}\frac{R_{ij}R_{ji}}{(\lambda_i - \lambda_j)^2}}_{\text{nonsinglet}}$$

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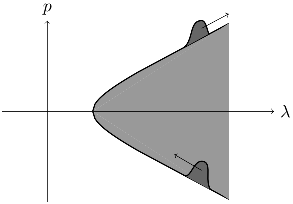
singlet — nonsinglet



↑
long strings

$L \to L + C$ amplitude from the Matrix Model

The long string state in MQM.

$$
\begin{aligned}
|w\rangle \quad &\equiv \quad \psi_{ij}(\lambda_1, \cdots, \lambda_N)|ij\rangle \\
&= \quad \begin{pmatrix} w(\lambda_1) & & \\ & w(\lambda_2) & \\ & & \ddots \end{pmatrix} \quad \psi_0(\lambda_1, \cdots, \lambda_N)
\end{aligned}
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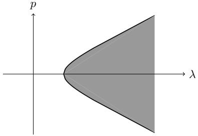
$$= \begin{pmatrix} w(\lambda_1) & & \\ & w(\lambda_2) & \\ & & \ddots \end{pmatrix} \psi_0(\lambda_1, \cdots, \lambda_N)$$

$w(\lambda)$ to be solved for [Fidkowski]

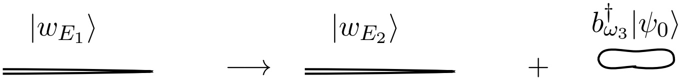
ground state of Fermi sea



It is a zero weight state, and is invariant under S'_N: $\lambda_i \leftrightarrow \lambda_j$ and $i \leftrightarrow j$.

$L \to L + C$ amplitude from the Matrix Model

We now compute



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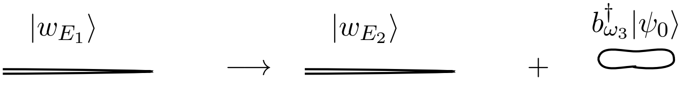

using the Born approximation:

$$\mathcal{A}^{\text{tree}}_{L\to L+C} = -2\pi i \langle w_{E_2}| b_{\omega_3} H'_{int} | w_{E_1}\rangle = \cdots$$

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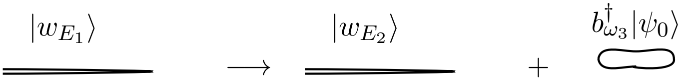

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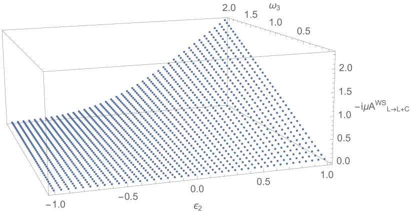
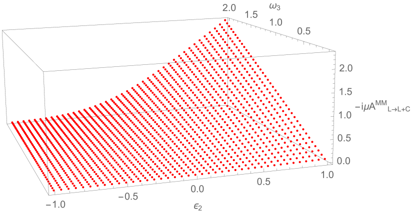
Results agree!



(a) Worldsheet



(b) Matrix Model

Future directions

- Understanding the MM dual of the FZZT brane itself. Add fundamental and anti-fundamental dofs to the matrix model. Is there a collective field theory one can write down?

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Thank you!